SEMINAR $(\infty, 2)$

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ABSTRACT. An *à la carte* menu of topics for a self-driven continuation of Seminar ∞.

CONTENTS

0. GENERAL COMMENTS

0.1. It's essentially impossible to learn this stuff if you're not excited about it. So excitement should be the primary consideration in determining which talks to give – or indeed, whether to participate in the first place.

0.2. Relatedly, it's essentially impossible to read such books as HTT=[20] and HA=[26] cover-to-cover in a productive way, or really even full chapters thereof. Rather, these references should be read selectively, with specific goals in mind.

0.3. These goals will probably not generally include "understanding all the details of the proofs". That's just the way of the world. This is probably quite a different approach to learning math from the standard undergraduate one (certainly from my own, at least), and a much more nuanced one. The proofs that end up mattering to you should eventually become at least intuitively clear in time.[1]

0.4. More specifically, you should generally skim through larger chunks of material searching for the core ideas and themes, and only then begin reading more carefully.[2]

0.5. Justin Noel has been running an ∞-categories seminar in Regensburg; feel free to check out his two course sites (here and here) for further ideas and inspiration.

0.6. It should go without saying that this list of references is surely extremely incomplete. You should also feel free to search around for other topics of interest.

0.7. This list is organized by topic, rather than by talks; I'd expect that many of the topics could span two or more talks, depending on the amount of detail given. I suggest that this generally remain flexible, as it has been through Seminar ∞ itself.

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[1] This is an instance of "backfilling", as described by Ravi Vakil here (towards the bottom in "On seminars").

[2] Lurie does a fantastic job of organizing his writing (e.g. each book has its own intro describing each chapter in broad strokes, each chapter has a more detailed intro describing what's accomplished in each section, and so on), which should be very helpful towards this end.

0.8. I've included purple asterisks to indicate that a talk might be somewhat more involved. These should not be taken too seriously, however, as it is only my own particular background that determines what I view as easy or hard.

1. Foundational material

This section describes some foundational topics. They can be useful for topics in other sections. However, in keeping with the overall message of §0, you should feel free to simply dive into the applications themselves and then return to these as necessary.

1.1. **Co/limits.** In [20, §4], there are a number of useful results on co/limits in ∞-categories (and their generalizations, (relative) Kan extensions) – in particular an ∞-categorical analog of Quillen's Theorem A.

1.2. **Presentable ∞-categories.** Most ∞-categories of lasting interest are presentable – spaces, spectra, et al. These are in a precise sense "presented by finite data". They contain all co/limits, and enjoy many other technically convenient properties which make them an extremely important tool in practice. The basics of presentable ∞-categories are introduced in [20, §5]. Moreover, the ∞-category $\mathcal{P}\mathrm{r}^{\mathrm{L}}$ of presentable ∞-categories and left adjoints admits a symmetric monoidal structure, with unit the ∞-category $\mathcal{S}$ of spaces. The ∞-category $\mathcal{S}\mathrm{p}$ of spectra is an idempotent commutative algebra, which determines its smash product symmetric monoidal structure. The algebra of $\mathcal{P}\mathrm{r}^{\mathrm{L}}$ is described in [26, §4.8].

1.3. **Stable ∞-categories.** These are the ∞-categorical analog of triangulated categories: the canonical example is the derived ∞-category of a ring, or more generally the derived ∞-category of a ring spectrum. Stability is merely a property, while triangulation is a structure; and yet the homotopy category of a stable ∞-category is canonically triangulated, where the distinguished triangles are the images of the cofiber sequences. This is all described in detail in [26, §1], and a brief overview that connects with some of the themes present in Seminar ∞ is given in [32, §0.3.1].

1.4. **(∞, n)-categories.** A *complete Segal space* is a simplicial space satisfying certain conditions; the ∞-category of these is equivalent to $\mathcal{C}\mathrm{at}_\infty$. More generally, one can define a "Segal space object" in a cartesian symmetric monoidal ∞-category $\mathcal{X}$, which models "$\mathcal{X}$-enriched ∞-categories" (and extra structure on $\mathcal{X}$ allows one to impose a completeness condition). One can iterate this to define (∞, n)-categories as certain n-fold simplicial spaces, i.e. as a full subcategory of $\mathrm{Fun}((\mathbf{\Delta}^{op})^{\times n}, \mathcal{S})$. This is described in [22, §1]. [also Θ_n-spaces as a model for (∞, n)-categories]

1.5. **Enriched ∞-categories.** For a general monoidal ∞-category $\mathcal{V}$ (whose monoidal structure is not necessarily cartesian), it is a much more subtle task to define $\mathcal{V}$-enriched ∞-categories. In [13], this is accomplished through the formalism of non-symmetric ∞-operads (so, parametrized over $\mathbf{\Delta}^{op}$ instead of over Fin_*).

2. Classical homotopy theory

This section describes some topics in classical homotopy theory that have been recast using ∞-categories.

2.1. **Rational and p-adic homotopy theory.** In [25], these classical ideas are reformulated in ∞-categorical terms.[3] Additionally, connections are drawn between rational homotopy theory and deformation theory (see §4.3). Power operations play a role in the p-adic story, as well.

2.2. **Goodwillie calculus.** [insert description here]

[3] However, intriguingly, there's a central result in p-adic homotopy theory (due to Mandell) for which Lurie admits to being unable to find a conceptual proof. This does not happen often: it seems that most of classical homotopy theory is truly ultimately formal.

2.3. **Algebra in the stable homotopy category.** Structured ring spectra are a special case of algebras in a symmetric monoidal ∞-category. The latter are studied throughout [26], while [26, §7] specializes to the study of the former. In particular, [26, §7.3] introduces the *cotangent complex formalism*, which describes the *tangent bundle* to an ∞-category and the corresponding *cotangent complex* functor $\mathbb{L}$. In the case of spaces, $\mathbb{L}$ gives the parametrized suspension spectrum; in the case of commutative ring spectra, $\mathbb{L}$ gives "topological André–Quillen homology" (TAQ); in the case of associative ring spectra, $\mathbb{L}$ gives "topological Hochschild homology" (THH).[4] This is also described from a sort of hybrid ∞-categorical/model-categorical point of view in [15].

2.4. **Thom spectra.** Classically, the Thom space of a vector bundle is the one-point compactification of the total space. This construction can also be spectrified. Thom introduced this construction in his thesis in order to compute the (G-structured) cobordism ring, which is the homotopy ring of the corresponding (G-structured) Thom spectrum. In the ∞-categorical formalism for Thom spectra introduced in [1, 2], a bundle over a space X gives a functor $X \to \mathrm{Mod}_R$ to the ∞-category of R-module spectra, and the Thom spectrum is simply a colimit. In [3] is proved "a simple universal property of Thom ring spectra" using this formalism.

2.5. **The cobordism hypothesis.** The $(n, n+1)$-dimensional cobordism ∞-category has objects the closed n-manifolds and morphisms the $(n+1)$-dimensional cobordisms between them. Allowing for manifolds with corners, we get an (∞, n)-category whose objects are 0-manifolds, 1-morphisms are 1-manifolds with boundary, 2-morphisms are 2-manifolds with corners, etc. Though quite geometric in nature, this admits a very algebraic universal property: it's freely generated by the fully dualizable object $\mathbb{R}^0$. This is outlined in [23].

2.6. **Chromatic homotopy theory*.** Chromatic homotopy theory organizes the ∞-category $\mathcal{S}\mathrm{p}$ in terms of its "thick subcategories", going back to the classical nilpotence and periodicity theorems [16]. This can be seen as providing a description of "Spec of the sphere spectrum" (or "Spec of $\mathbb{F}_1$") via categorified algebraic geometry – that is, the algebraic geometry of presentably symmetric monoidal ∞-categories – as described in [32, §0.3.2].

2.7. **Elliptic cohomology.** As described in [30], in chromatic homotopy theory, "level 0" corresponds to rational cohomology, "level 1" corresponds to complex K-theory, and "level 2" corresponds to elliptic cohomology. The survey [21] gives an expository account of the higher-algebraic point of view on elliptic cohomology (and in particular the cohomology theory tmf of topological modular forms), which area forges deep connections between homotopy theory and arithmetic geometry / number theory. In particular, it takes the novel perspective that tmf should be constructed as the global sections not of a spectral sheaf on the moduli stack of ordinary elliptic curves, but as the global sections of the structure sheaf of the moduli stack of spectral elliptic curves.

3. Factorization homology

Factorization homology is the procedure of "gluing" the values of an $\mathbb{E}_n$-algebra A over an n-manifold M, giving an object $\int_M A$. When $M = S^1$, this recovers topological Hochschild homology $\mathrm{THH} := \int_{S^1}$, which computes the derived functor of abelianization for associative algebras (as indicated in §2.3). This interface between manifold topology and higher algebra is productive for the study of both, and admits connections with mathematical physics and algebraic geometry.

3.1. **Homology theory for manifolds.** A "homology theory for (finite) spaces" is a functor $H : \mathcal{S}^{\mathrm{fin}} \to \mathcal{C}$ satisfying the Eilenberg–Steenrod axioms, notably excision: it takes pushouts of spaces to pushouts in the (generally stable) ∞-category $\mathcal{C}$. Such a functor is uniquely determined by its value on $\mathrm{pt} \in \mathcal{S}^{\mathrm{fin}}$: categorically, this corresponds to the fact that finite spaces can be built by a finite number of gluings-on of the terminal object (i.e. cells-and-disks constructions of CW complexes). A "homology theory for (finitary) manifolds" is a functor $H : \mathcal{M}\mathrm{fld}_n \to \mathcal{C}$ (where now $\mathcal{C}$ need not

[4] For more on THH, see §3.

be stable) satisfying a *multiplicative* version of excision: it takes a collar-gluing of n-manifolds, say $M = M_1 \amalg_{N\times\mathbb{R}} M_2$ to a *tensor product*, i.e. we require $H(M) \simeq H(M_1) \otimes_{H(N\times\mathbb{R})} H(M_2)$. Now, n-manifolds are built up from $\mathbb{R}^n$, and once again such a functor is uniquely determined by its value on $\mathbb{R}^n \in \mathcal{M}\mathrm{fld}_n$. But whereas the object $\mathrm{pt} \in \mathcal{S}^{\mathrm{fin}}$ has no interesting structure, the object $\mathbb{R}^n \in \mathcal{M}\mathrm{fld}_n$ is an $\mathbb{E}_n$-algebra, and we must remember the value of $H(\mathbb{R}^n)$ as such. This is described in [4], and is also reiterated in [31, §2].

3.2. **Nonabelian Poincaré duality.** Grouplike $\mathbb{E}_n$-algebras are equivalent to based n-connected spaces, via the de/looping adjoint equivalence $B^n : \mathrm{Alg}_{\mathbb{E}_n}(\mathcal{S}) \simeq \mathcal{S}_*^{\geq n} : \Omega^n$. The factorization homology of $A \in \mathrm{Alg}_{\mathbb{E}_n}(\mathcal{S})$ can be redescribed as $\int_M A \simeq \mathrm{map}^c(M, B^n A)$, the compactly-supported mapping space. When A is an abelian group, this recovers ordinary Poincaré duality by applying π_* (and using the Dold–Thom theorem for the homotopy groups of labeled configuration spaces). This result is classical, going back to "scanning map" constructions of Salvatore, Segal, and McDuff. Through the characterization of factorization homology of §3.1, this can be proved in a completely functorial manner – no longer requiring any choices such as ε-balls in a manifold. This is described in [4, §4], and is also reiterated in [31, §2.4].

3.3. **Poincaré/Koszul duality*.** For more general symmetric monoidal ∞-categories than $\mathcal{S}$ – specifically, ones which are not necessarily cartesian symmetric monoidal – factorization homology no longer obeys such an immediate form of nonabelian Poincaré duality. Rather, it must be intertwined with Koszul duality in the algebra variable. However, this alone only recovers an equivalence under connectivity hypotheses; to remove these, one must instead use a certain non-affine lift of Koszul duality, which takes place in the context of deformation theory (see §4.3). This is described in [5], and is also reiterated in [31, §3] (with a particular emphasis on the deformation-theoretic perspective). This is based in the ∞-category $\mathcal{ZM}\mathrm{fld}_n$ of *zero-pointed n-manifolds*, which is "bivariant" in that its morphisms account for both embeddings and collapse maps among n-manifolds; this is introduced in [6] (though reviewed adequately in the aforementioned references).

3.4. **Factorization homology and higher algebra*.** Factorization homology is connected with a number of topics in higher algebra, which are described in [26, §5], e.g.: Dunn additivity ($\mathbb{E}_m \otimes \mathbb{E}_n \simeq \mathbb{E}_{m+n}$, which implies that $\mathrm{Alg}_{\mathbb{E}_m}(\mathrm{Alg}_{\mathbb{E}_n}(\mathcal{C})) \simeq \mathrm{Alg}_{\mathbb{E}_{m+n}}(\mathcal{C})$); Koszul duality; higher centers (i.e. generalizing the usual center construction for monoids and groups); Verdier duality.

3.5. **Factorization homology of higher categories**.** In the above-described formulation of factorization homology, one labels n-disks in an n-manifold by elements of an $\mathbb{E}_n$-algebra. Just as a monoid can be thought of as a one-object category, so can an $\mathbb{E}_n$-algebra be thought of as a one-object (∞, n)-category. Factorization homology can be likewise generalized: we now study *disk-stratifications* of our n-manifold M, and for an (∞, n)-category $\mathcal{C}$ we can label the k-dimensional strata of M by k-morphisms in $\mathcal{C}$, so that the source and target data in $\mathcal{C}$ align with the geometry of the disk-stratification. This is described in [7].

4. Derived geometry

In classical algebraic geometry, schemes are locally built from commutative rings: for a commutative ring A, $\mathrm{Spec}(A)$ is the affine scheme whose ring of global functions is A. This construction defines a contravariant embedding $\mathrm{Spec} : \mathrm{CRing}^{op} \hookrightarrow \mathrm{Sch}$ which (being a right adjoint) commutes with limits, so that e.g. the intersection of affine schemes is computed by the pushout of commutative rings: $\mathrm{Spec}(A) \times_{\mathrm{Spec}(B)} \mathrm{Spec}(C) \cong \mathrm{Spec}(A \otimes_B C)$. However, just as in manifold theory, this does not give the desired answer if the intersection is not transverse. This is repaired by passing to a homotopical setting and taking a *derived* tensor product.

This derived enhancement of algebraic geometry can be enacted using (a model categoriy of) simplicial commutative rings, or even commutative differential graded algebras if one is working in characteristic 0, but one can also pass to the more flexible setting of $\mathbb{E}_\infty$-ring spectra. The question of which choice makes the most sense depends on the desired application: for applications in algebraic geometry it generally suffices to work with SCR's or cdga's, but these only encompass $H\mathbb{Z}$-module spectra and so miss most of the interesting spectra that arise in homotopy theory. The term *derived* (or in the old days, *homotopical*) *algebraic geometry* can refer to either the SCR/cdga- or spectrally-flavored variants, whereas the term *spectral algebraic geometry* of course refers specifically to the latter.

Besides taking intersections, another ubiquitous construction in geometry is the formation of colimits, especially quotients by group actions. These likewise do not generally give the desired answer if the group action isn't free, and they can likewise be repaired by taking derived quotients, which will in general give rise to a *stack*: a (1- or ∞-)groupoid-valued functor that satisfies an appropriately homotopical version of the sheaf condition (see §7.1). The 1-groupoid-valued variant have long played an important role in classical algebraic geometry, but of course various categorical constructions naturally leave this realm.

Derived methods are also useful in *differential* geometry; see §4.6.

4.1. **Derived algebraic geometry.** This was actively developed most notably by Toën–Vezzozi for a long time in the language of model categories, although they eventually updated their methods to the much more flexible setting of ∞-categories, see e.g. [36] and the many references therein. Lurie's thesis [18] also serves as a very nice introduction.

4.2. **Spectral algebraic geometry.** Most of the work done in derived algebraic geometry takes place over a field of characteristic 0, because this is much easier than even working over a field of positive characteristic. By the time you pass all the way to the sphere spectrum, things take on quite a different flavor. This has mainly only been developed by Lurie. Much of his work on this topic is contained in his new (and as yet unfinished) book [28], but see also §2.7.

4.3. **Deformation theory.** Deformation theory is the local study of stacks: the algebro-geometric object that encodes "an infinitesimal neighborhood of a point in a stack" is called a *formal moduli problem*. In characteristic 0, the ∞-category of these is equivalent to that of differential graded Lie algebras: the latter gives the "tangent complex" of the former (with the Lie bracket picking up "the bracket of vector fields"). This is described expositorily in [19], and carefully in [24].

4.4. **The Galois group of a stable homotopy theory.** Another approach to studying derived algebraic geometry is by replacing geometric objects (schemes, stacks, etc.) by their ∞-categories of quasicoherent sheaves (roughly, "vector bundles"). In the paper [29] is studied the "Galois group" (a geometric flavor of fundamental group) of such an ∞-category, including various particular ones of interest in stable homotopy theory.

4.5. **Shifted symplectic and Poisson structures*.** In ordinary differential geometry, symplectic and Poisson structures on manifolds are a very active area of study, and connect with mathematical physics in various ways. On the other hand, recent advances in physics suggest that the appropriate objects should actually be of a derived nature, which leads naturally to a desire for derived versions of symplectic and Poisson structures. Once there are "cohomological" directions around, one can also consider "shifted" variants of these: whereas an ordinary symplectic structure is the data of an isomorphism between the tangent bundle and the cotangent bundle, a shifted symplectic structure is the data of a quasi-isomorphism between the tangent complex and a shift of the cotangent complex. This is surveyed in [34], while the material is originally introduced in [33, 12].

4.6. **Synthetic differential geometry and cohesion.** A topos (see §7) is called *cohesive* if it carries additional structure that witnesses its objects as something like "sets with extra structure", e.g. manifolds or topological spaces. A cohesive ∞-topos is analogous, only now the "underlying" objects are not 0-groupoids but ∞-groupoids. In [35] is given a comprehensive treatment of abstract cohesive ∞-topoi as well as the specific ones corresponding to various notions in differential geometry, including smooth manifolds, smooth supermanifolds, and their infinitesimal analogs. Many applications in mathematical physics are also described – this is the underlying motivation of the work. The purely mathematical story is surveyed in [35, §1.2], which gives references to later sections as it describes their contents.

5. Equivariant homotopy theory

It has long been understood that a good theory of equivariant homotopy theory doesn't just keep track of a homotopical action (i.e. a functor $BG \to \mathcal{S}$ of ∞-categories): rather, one should keep track of the fixedpoint data for all subgroups of G. However, this ends up being equivalent to $\mathrm{Fun}(\mathrm{O}_G^{op}, \mathcal{S})$, the ∞-category of presheaves on the *orbit category* of G, whose objects are transitive G-sets and whose morphisms are G-equivariant functions: for an object X, we think of the space $X(G/H)$ as "the genuine H-fixedpoints of X" (and denote it by X^H). From here, we can also form a "genuine" version of G-spectra: these have deloopings not just for spheres S^n, but for representation spheres S^V (i.e. the one-point compactification of a G-representation V).

5.1. **Epiorbital categories.** It turns out that the relevant features of the orbit category O_G could be abstracted into a more general framework: this is an example of an *epiorbital category.* This allows for a generalization from $n = 2$ to arbitrary values of n of a certain coincidence between n-polynomial functors $\mathcal{S}\mathrm{p} \to \mathcal{S}\mathrm{p}$ (in the sense of Goodwillie calculus) and certain genuine equivariant spectra. This is described in [14].

5.2. **Parametrized homotopy theory.** The study of equivariant homotopy theory through $\mathcal{C}\mathrm{at}_\infty$-valued presheaves on epiorbital categories (or equivalently but more technically usefully, co/cartesian fibrations thereover) has been undertaken in a series of papers, beginning with [10] (and see the references given therein for more).

5.3. **Spectral Mackey functors.** [insert description here]

6. Algebraic K-theory

Algebraic K-theory was classically introduced as an invariant of rings (or more generally "Waldhausen categories"), an algebro-geometric analog of K-theory for spaces. While it's been an important branch of math for many decades and admits deep connections to a wide variety of fields, its moral underpinnings – the reason for its prominence – have been poorly understood for most of its existence. This situation has been substantially improved with the advent of ∞-categorical techniques.

6.1. **Barwick's characterization.** In [9] is given a direct generalization of the construction of algebraic K-theory from Waldhausen categories to Waldhausen ∞-categories. The ∞-category $\mathcal{W}\mathrm{ald}_\infty$ of these admits a certain "derived" variant $\mathscr{D}_{\mathrm{fiss}}(\mathcal{W}\mathrm{alg}_\infty)$, through which the K-theory functor factors and moreover acquires the universal property $K \simeq P_1(\iota)$, i.e. it's the linear approximation to the "maximal subgroupoid" functor ι.

6.2. **Blumberg–Gepner–Tabuada's characterization.** In [11] is given a different universal characterization of algebraic K-theory, this time restricted to stable ∞-categories and of a "motivic" flavor.

6.3. **Rotation invariance*.** In a stable ∞-category $\mathcal{C}$, the cofiber sequence $X \to 0 \to \Sigma X$ implies that we obtain an equality $[X] = -[\Sigma X]$ in the Grothendieck group $K_0(\mathcal{C})$. It follows that $[X] = [\Sigma^2 X]$, naturally for all objects $X \in \mathcal{C}$. In [27] is described a more primordial source of this equality, namely the "rotation invariance" of algebraic K-theory.

6.4. **The cyclotomic trace from factorization homology.** Algebraic K-theory admits a trace map $K \to THH$: viewing THH as functions on the free loopspace, this heuristically takes a vector bundle to the function taking a loop to the trace of the corresponding monodromy endomorphism. This is invariant under rotation of the circle, which gives a factorization $K \to TC$ to *topological cyclic homology*. This *cyclotomic trace* map ends up being a very important tool in studying algebraic K-theory, as it is "locally constant" (in the sense of Goodwillie calculus). This was originally constructed using point-set methods in genuine-equivariant homotopy theory, which largely obscures the true nature of this crucial construction. In [8] is given a construction of the cyclotomic trace purely through universal properties, using nothing but Goodwillie calculus and the geometry of 1-manifolds; factorization homology of enriched ∞-categories (as in §3.5) is employed as an organizing tool.

7. Higher topos theory and its applications

Ordinary topoi are useful both in geometry – as categories of sheaves – and in logic – as "systems of logic" (e.g. ones in which the law of the excluded middle may not hold). At least the former is continues to be a prevalent notion in higher-categorical contexts. Here, when the target is no longer just a 1-category, the sheaf condition is replaced by a homotopical variant, in which the equalizer becomes a totalozation (i.e. a cosimplicial limit). As with 1-topoi, ∞-topoi can be characterized in a number of complementary ways, and they admit an "internal homotopy theory" which behaves like that of spaces (the terminal ∞-topos: ∞-sheaves on a point) in many ways.

7.1. **Sheaves, stacks, and internal homotopy theory.** These are discussed thoroughly in [20, §6]. There is also a "Cliff's Notes" version [17] of HTT (which predates it), which focuses on ∞-topoi.

7.2. **Connections with topological spaces.** A topological space gives rise to a locale, its category of open sets: this is its topos-theoretic avatar, and taking ∞-sheaves gives an ∞-topos. This construction is the arena of various homotopical approaches to point-set topology, e.g. shape theory and dimension theory. This is discussed in [20, §7].

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