SYLLABUS: MULTIVARIABLE CALCULUS (MATH 226)
SECTION 39565R
FALL 2018

Contents

1. General information

Instructor.
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Teaching assistant (TA).
name: Jayson Grassi
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office hours: Mon. 10-11am and Tues. 8-10am in KAP 263

Course logistics.
lectures: MWF at 11-11:50am in GFS 118
discussion sections: TuTh at 12-12:50pm or 1-1:50pm in THH B10
textbook: *Essential Calculus*, Stewart (2nd ed.)
course website: etale.site/teaching/f18-226/
Blackboard: yes

Grades. Your final grade in the course will be determined by your performance on your homework, quizzes, and exams. Your scores on these will be combined into a single overall *raw score*, according to the following weights:

homework: 15%, quizzes: 10%, midterms: 2x20%, final exam: 35%.

last updated: September 5, 2018

[1]These office hours are tentative, and are subject to change.

[2]If all of my office hours conflict with your schedule, I am also available to meet by appointment.

At the end of the semester, letter grades will be assigned on a curve based on these raw scores, with the top quartile receiving grades in the A-range and the second quartile receiving grades in the B-range (so the median grade will be the cutoff between a B- and a C+).[3] Over the course of the semester, I will periodically compute your raw scores up to that point and inform you of the current grade distribution, to give you a sense of where you stand.

Homework. Homework will be assigned each Monday, and will be due at the beginning of discussion section on the following Tuesday (i.e. 8 days later). Late homework will not be accepted. An assortment of the problems from each homework will be graded. In order to account for unexpected circumstances, your lowest two homework scores will be dropped.

Quizzes. You will be given a quiz at the beginning of each Tuesday's discussion section, which will cover the material on the homework that is due that same day. There will be no quiz the first week. In order to account for unexpected circumstances, your lowest two quiz scores will be dropped.

Exams. The exam schedule is as follows:

midterm 1: Wed. 9/26, midterm 2: Wed. 10/31, final exam: Wed. 12/5 at 2-4pm.[4]

There are no make-up exams. In particular, in accordance with university policy, there will be a single final exam that all Math 226 students in all sections will take simultaneously; it is not possible to take the final exam at any other time, and if you cannot take it at this time then you may have to receive a grade of Incomplete.

Reading. Reading the textbook is *required*. This will not be explicitly evaluated, but it will be essential for staying on top of the material.

2. Course description

General description. Calculus is the mathematics of *change*. In particular, it allows us to understand and quantify *how fast* things are changing (via differentiation), as well as *how much* things are changing (via integration).

So far, you have studied calculus in one variable. However, most real-life problems have many variables. In this class, we will extend the techniques of single-variable calculus to the multivariable setting, such as surfaces in 3-dimensional space.

The course will culminate with *three different generalizations* of the fundamental theorem of calculus: Green's theorem, Stokes's theorem, and the divergence theorem.[5] These

[3]The curve will include students who have dropped the course (who will presumably lie towards the bottom of the rankings).

[4]The midterms will be given in class, while the final exam will be given in a location that is TBD.

[5]In fact, these three theorems are all special cases of a single mega-theorem, the *generalized Stokes's theorem*, which is beyond the scope of this course.

theorems are really fantastic – true mathematical gems, and certainly the highlight of the entire calculus sequence. I'm excited to share them with you; I hope you're excited, too.

Chapter-by-chapter description. We will cover the last four chapters of the book, which can be summarized as follows.

Chapter 10: Vectors and the geometry of space. In this chapter, we will learn how to think in 2- and 3-dimensional space. This will lay crucial groundwork for us, whose importance cannot be overstated. For instance, just as the graph of a one-variable function defines a curve in 2-space, the graph of a two-variable function defines a surface in 3-space.

Chapter 11: Partial derivatives. In this chapter, we will study derivatives in the multivariable setting. Recall that in single-variable calculus, the derivative of a function $f(x)$ tells us how fast that function is changing as we change the input variable x; this corresponds to the instantaneous slope of the function. But for a function of two variables $g(x, y)$, the rate of change of the function depends on how we change the input variables x and y! For instance, the function may increase as x increases, but decrease as y increases. If so, when we simultaneously increase x and y, these two effects will be in tension: the function might increase or might decrease, depending on how much we're changing each variable and also on how much $g(x, y)$ changes as each variable changes. Amazingly, however, we will see that it is possible to efficiently package all of these various rates of change into a single mathematical object. We will also study all of this geometrically, in terms of the surface in 3-space that is the graph of $g(x, y)$.

Chapter 12: Multiple integrals. In this chapter, we will study integrals in the multivariable setting. Recall that in single-variable calculus, the integral

$$\int_R f(x)\ dx$$

measures the 2-dimensional area bounded between the graph of $f(x)$ and the x-axis over the region R of the x-axis. Here, we define double and triple integrals. The double integral

$$\iint_S g(x, y)\ dx\ dy$$

measures the 3-dimensional volume bounded between the graph of $g(x, y)$ and the xy-plane over the region S in the xy-plane. Can you guess what the triple integral

$$\iiint_T h(x, y, z)\ dx\ dy\ dz$$

is computing?

Chapter 13: Vector calculus. It all comes together in this chapter, in which we will study the three aforementioned generalizations of the fundamental theorem of calculus.

3. Comments, suggestions, and resources

General comments and suggestions. In my experience, the passage from single-variable calculus to multivariable calculus is at least as much of a conceptual leap as the passage from precalculus to single-variable calculus. As indicated above in the description of Chapter 12, this class will truly stretch your mind. Moreover, there is a lot of material to cover, and as a result the course goes very quickly.

In view of all of this, you should expect to spend at least as much time on this course as you have ever spent on a math class, and quite possibly more. The only way to really get to understand new and challenging mathematical concepts is to spend *a lot* of time with them.

In addition, I encourage you to take advantage of the resources available to you that are described below, *throughout* the semester (i.e. not just in the days leading up to exams). These resources can make the time that you do spend on this course much more effective, especially if you use them wisely.

I like to compare mathematics with running. The only way to get really good at running is to do a lot of it. Moreover, you have to do a lot of it *for a long time*. You wouldn't start training for a marathon just a few days before – that'd be way too late! Similarly, you should not expect to do well on the exams if you don't stay on top of the material throughout the course.

It is worth mentioning that the material in Chapter 10 may overlap with topics that you have seen in high school or in other college-level courses. However, I urge you to pay close attention nonetheless: even if the material is familiar, it is quite likely that you are not yet as familiar with it as you will need to be in order to succeed in the later parts of the course. By its very nature, math is incredibly self-referential: each new idea is built on a long progression of ideas that must come before it.[6,7] If you don't have a very solid grasp on Chapter 10, you will end up struggling with the later material.

With all of this said, it is also worth mentioning that this is a very fun and rewarding course, which is in many ways quite different from what you have seen in the first two semesters of calculus.

Attribution. Whenever you receive help on any of your homework problems (such as in the ways described below – office hours, Math Center, collaboration, etc.), you must indicate this on your homework before turning it in. For instance, at the top of your homework you might write "I worked on my homework with Alice and Bob", or next to problem 17 you might write "I found a hint/solution to this on mathhelp4u.blogspot.com". *It is considered cheating if you do not provide such attributions*, and you may face consequences accordingly.

[6]This makes it hard for mathematicians to describe their research, as opposed to e.g. a biologist who can simply say "I study cancer", or an astronomer who can simply say "I study black holes".

[7]If you can remember it, try to think of all the auxiliary concepts that go into the definition of a *limit*. There are a lot of them! (Remember when you first learned about real numbers?)

Office hours. Almost certainly, the most valuable and effective resource for you will be my office hours. You should know, however, that they are made far more valuable if you spend time with the material *before* coming to office hours.

Compare this to learning a musical instrument. There are two aspects to this: taking lessons from a teacher, and practicing on your own. These are both very helpful. A teacher can give you suggestions for improving your technique, and broader guidance in your musical studies. But this is useless if you don't practice on your own as well.

On the other hand, you should not feel that you need to come to office hours with specific questions prepared. When we are confused, often the hardest part of becoming un-confused is just articulating exactly where our confusion lies. Having spent many years learning and teaching (and researching) math, I will very likely be able to help you identify your confusion – and then resolve it.

Note that if you do not work on the homework before coming to office hours, *you won't even know whether you're confused*. You will still be welcome to come to office hours in this case, but this will make that time much less effective for you.

Math Center. The Math Center (KAP 263) is an amazing resource: in essence, it is a centralized location where students can come get help from TAs, that is open pretty much all day every (week)day.[8] Your TA holds office hours there, as do all other TAs for core math classes – and they are all there to help you.

The schedule of office hours will be fixed by the end of the second week of classes, at which time it will be available on the Math Center website. In particular, it will list the office hours of all TAs for Math 226. They are instructed to help any and all Math 226 students who show up (this means you!). Since all of the TAs naturally have different teaching styles, you should feel free to go to different TAs' office hours to see whose help you find most effective.

Moreover, while the other TAs working at the Math Center are instructed to prioritize their own topic first (e.g. Math 125, Math 126, Math 226, etc.), they are also instructed to work with any students who need help. So especially when it's not exam season, you should be able to get math help pretty much all day every (week)day.

Collaboration. Collaboration with your fellow students can be a very effective way of learning, and is highly encouraged. Often, we may think we understand something very well, but in trying to explain it to somebody else we find the gaps in our understanding.[9]

[8]Towards helping you appreciate this, it's worth mentioning that it is fairly unique to USC. For instance, at my undergrad institution we had a similar sort of math center, but it was only open 8 hours per week.

[9]There was some research that came out about this recently. Subjects were asked to rate their understanding of some everyday mechanism (e.g. a ballpoint pen or the flushing mechanism of a toilet). Then, they were asked to explain it. Then, they were asked to rate their understanding again. As you might imagine, their own estimation of their understanding dropped significantly from the first round of rating to the second. I believe that the *single most important skill* that a human being can obtain from doing math

However, *everyone must write up their own solutions separately*: you may not copy each other's work. Moreover, as described above, you must indicate on your homework if you worked with anyone else.

Extra problems. You should view the homework as the *minimum* set of problems that you should work through: it is highly encouraged to work through other problems as well. The problems at the end of each section are conveniently organized in groups: a group of easy problems, a few groups of mid-level problems, and then a group or two of hard problems. So for instance, if you are struggling with the concepts, you should test yourself on some easy problems to try and figure out what you're missing. And if you are studying for an exam and want to really challenge your grasp of the material, you should test yourself on some hard problems.

I believe that it can be extremely worthwhile to *do the same problems multiple times*, even at the expense of seeing fewer problems overall. In preparing for exams, students often make a point of doing lots of different problems, with the idea that this will make them more likely to have already seen whatever problems end up on the exam (perhaps with the numbers changed around). However, it's not very useful to have seen a problem if you don't have a solid understanding of how to solve it! Understanding is not binary: there are degrees to it, and it can ebb and flow with time. In my experience, it is generally a better idea to become very familiar with a smaller list of problems by doing each of them many times.[10]

The book. Reading a math textbook is very different from reading a novel. You should expect to have to read things slowly, and often multiple times, in order to fully understand what is being expressed. If you are getting confused, you may even want to take a break from reading the book and come back to it again later. Keep in mind that the book was written *for you*: by and large, it is as clear as it can possibly be. The reason that it is difficult to read is because the material itself is difficult. And that's okay – it's as it should be.

The internet. There are tons of resources out there, and some of them are very good.

4. Miscellanea

Blackboard. Homework assignments, readings, and announcements concerning the course will be posted to Blackboard; it is your responsibility to check there frequently. Your scores will also be recorded on Blackboard; it is your responsibility to check that your scores are recorded correctly.

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is a deep appreciation of what "understanding" really means. It could probably be argued that many of the world's problems ultimately result from people being overly confident in their own level of understanding.

[10]This level of familiarity will also allow you to see deeper similarities between problems, which you may not have appreciated if you had only skimmed through all the different problems.

Course website. As the course progresses, various materials (such as quiz solutions) will be available at the course website.

Communication. In general, you are always welcome to email me and/or the TA about any issue regarding the course. However, please check the syllabus and the Blackboard announcements before emailing, in case your question has already been answered.

Calculators. We will not be using calculators in this course.

Classroom conduct. Please remember that you and your classmates are here to learn, and refrain from disruptive or otherwise inconsiderate behavior. In particular, cell phones are to be turned off and kept out of sight during class: even if you yourself are able to multitask or already understand the material, using your phone will likely be distracting to your fellow classmates. If I see you on your phone, I will ask you to either put it away immediately or leave class until you are done with it.

DSP accommodations. Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. Please ensure that DSP delivers its letter of verification to me as soon as possible, to give me enough time to make suitable arrangements. *I must receive your letter at least two weeks before an exam in order to guarantee that you will receive the corresponding accommodations.* You can find information about DSP (location, hours, contact, etc.) here.

Academic integrity. You must abide by the university policies on academic integrity, which you can review here. In essence, these policies require you to be honest. So, please: be honest.

Revisions to this syllabus. This syllabus is subject to minor changes throughout the semester, as needed. In the interest of transparency, all versions will remain available on the course website.