Math 425 worksheet

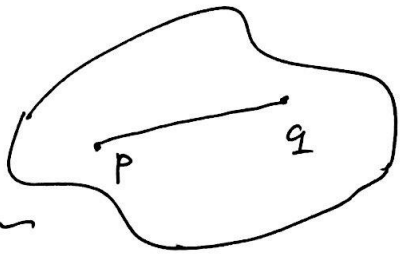
October 23rd

1. Prove that every convex subset $E \subset \mathbb{R}^n$ is path-connected.

conti

$\forall p, q \in E$. WTS there is a path $f: [a,b] \to E$ s.t. $f(a) = p$, $f(b) = q$.

Since E is convex, every point between p and q is in E. Take the path to be the line segment from p to q. $f: [a,b] \to E$ conti



$f(a) = p$

$f(b) = q$.

$\boxtimes$

2. (a) Show that $\mathbb{R}$ and $\mathbb{R}^n$ are not homeomorphic if $n \geq 2$.
(b) Show that $(0,1)$ is not homeomorphic to $(0,1]$.

(a) $\mathbb{R} \setminus \{0\} \not\cong \mathbb{R}^n \setminus \{f(0)\}$.

↑ disconnected ↑ connected.

(b) If $[0,1] \cong (0,1)$.

remove 1 from $(0,1]$, then

$(0,1] \setminus \{1\} \cong (0,1) \setminus \{f(1)\}$.

3. Let (M, d) be a metric space. Show that any finite set S with at least two elements is disconnected.

$S \subset M$. $|S| \geq 2$.

pick $x \in S$. then $\{x\}$ is a proper subset of S.

$\{x\}$ is closed in S b/c it's a singleton.

$S \setminus \{x\}$ is closed in S b/c

$S \setminus \{x\} = \{p_1, p_2 \cdots p_n\} = \bigcup_{i=1}^{n} \{p_i\}$.

finite union of closed sets is closed in S.

$(\{x\}) \cup (S \setminus \{x\}) = S$.

↑ closed nonempty open in S

↑ closed. nonempty open in S.

$\Rightarrow S$ is disconnected. ⊠