Math 226 – Midterm 2 Name: ______________________

Show all your work, and indicate clearly if you continue on the back. The entire exam is worth 50 points.

(4 points) 1. Find an equation of the tangent plane to the surface $z = \ln(x - 2y)$ at the point $(3, 1, 0)$, and use it to approximate the z-value of the surface at which $(x, y) = (2, \frac{1}{2})$.

(8 points) 2. If $z = f(x, y)$ where $x = r^2$ and $y = 2rs$, find $\frac{\partial^2 z}{\partial r \partial s}$.

(8 points) 3. The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point $(1, 2, 2)$ is $120°$. Find the rate of change of T at $(1, 2, 2)$ in the direction toward the point $(2, 1, 3)$.

(8 points) 4. Find the absolute extrema of the function $f(x, y) = x^2 + y^2 + x^2 y + 4$ over the domain

$$D = \{(x, y) \in \mathbb{R}^2 \mid |x| \leq 1 \ , \ |y| \leq 1\} \ .$$

(8 points) 5. Using Lagrange multipliers, find the points on the surface $y^2 = 9 + xz$ that are closest to the origin.

(6 points) 6. Consider the solid cube in $\mathbb{R}^3$ defined by $0 \leq x, y, z \leq 4$. If the cube has charge density $\rho(x, y, z) = xy$, use the midpoint rule with $l = m = n = 2$ to approximate its total charge.

(8 points) 7. Consider the solid cylinder $E \subset \mathbb{R}^3$ defined by $x^2 + y^2 \leq 1$ and $0 \leq z \leq 1$. If the cylinder has density function $\rho(x, y, z) = (x^2 + y^2)z$, find its center of mass.