Solutions: quiz 10, discussion section 10am

Math 226, Fall 2019, Prof. Mazel-Gee

1. The region E may be described as the spherical wedge

$$E = \left\{ (\rho, \theta, \phi) \in \mathbb{R}^3 \mid \rho \in [0,1] \ , \ \theta \in \left[-\tfrac{\pi}{4}, \tfrac{3\pi}{4} \right] \ , \ \phi \in \left[0, \tfrac{\pi}{2} \right] \ \right\} \ .$$

Hence, we may evaluate the integral in spherical coordinates as

$$\begin{aligned}
\iiint_E z \ dV &= \int_{\phi=0}^{\phi=\pi/2} \int_{\theta=-\pi/4}^{\theta=3\pi/4} \int_{\rho=0}^{\rho=1} (\rho \cos \phi) \cdot \rho^2 \sin \phi \ d\rho \ d\theta \ d\phi \\
&= \left(\int_{\phi=0}^{\phi=\pi/2} \sin \phi \cdot \cos \phi \ d\phi \right) \cdot \left(\int_{\theta=-\pi/4}^{\theta=3\pi/4} d\theta \right) \cdot \left(\int_{\rho=0}^{\rho=1} \rho^3 \ d\rho \right) \\
&= \left(-\frac{1}{4} \cos 2\phi \right) \Bigg|_{\phi=0}^{\phi=\pi/2} \cdot \pi \cdot \frac{1}{4} = \left(-\frac{1}{4} \cdot ((-1) - 1) \right) \cdot \frac{\pi}{4} = \frac{\pi}{8} \ .
\end{aligned}$$

2. We rewrite the equation as $x^2 + y^2 + z^2 = 2x$, which we then rewrite as $(x-1)^2 + y^2 + z^2 = 1$. This is a sphere of radius 1 centered at the point $(x, y, z) = (1, 0, 0)$.