Solutions: quiz 10, discussion section 11am

Math 226, Fall 2019, Prof. Mazel-Gee

1. For the point $A = (x, y, z) = (1, 1, \sqrt{6})$, we compute that

$$\begin{aligned}
\rho &= \sqrt{x^2 + y^2 + z^2} = \sqrt{1 + 1 + 6} = 2\sqrt{2} \ , \\
r &= \sqrt{x^2 + y^2} = \sqrt{1 + 1} = \sqrt{2} \ , \\
\theta &= \arctan\left(\frac{y}{x}\right) = \arctan(1) = \frac{\pi}{4} \ , \\
\phi &= \arccos\left(\frac{z}{\rho}\right) = \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \ ,
\end{aligned}$$

so that in spherical coordinates $A = (\rho, \theta, \phi) = (2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{6})$ and in cylindrical coordinates $A = (r, \theta, z) = (\sqrt{2}, \frac{\pi}{4}, \sqrt{6})$. For the point $B = (x, y, z) = (3, 4, 5)$, we compute that

$$\begin{aligned}
\rho &= \sqrt{x^2 + y^2 + z^2} = \sqrt{9 + 16 + 25} = 5\sqrt{2} \ , \\
r &= \sqrt{x^2 + y^2} = \sqrt{25} = 5 \ , \\
\theta &= \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{4}{3}\right) \ , \\
\phi &= \arccos\left(\frac{z}{\rho}\right) = \arccos\left(\frac{1}{\sqrt{2}}\right) \ ,
\end{aligned}$$

so that in spherical coordinates $B = (\rho, \theta, \phi) = (5\sqrt{2}, \arctan(\frac{4}{3}), \arccos(\frac{1}{\sqrt{2}})$ and in cylindrical coordinates $B = (r, \theta, z) = (5, \arctan(\frac{4}{3}), 5)$.

2. This region $E \subset \mathbb{R}^3$ is the spherical wedge

$$E = \left\{(\rho, \theta, \phi) \in \mathbb{R}^3 \mid \rho \in [0, 2] \ , \ \phi \in \left[0, \tfrac{\pi}{3}\right]\right\} \ .$$

So, we may compute its volume as

$$\begin{aligned}
\iiint_E 1 \ dV &= \int_{\phi=0}^{\phi=\frac{\pi}{3}} \int_{\theta=0}^{\theta=2\pi} \int_{\rho=0}^{\rho=2} \rho^2 \sin\phi \ d\rho \ d\theta \ d\phi \\
&= \left(\int_{\phi=0}^{\phi=\frac{\pi}{3}} \sin\phi \ d\phi\right) \cdot \left(\int_{\theta=0}^{\theta=2\pi} d\theta\right) \cdot \left(\int_{\rho=0}^{\rho=2} \rho^2 \ d\rho\right) \\
&= \frac{1}{2} \cdot 2\pi \cdot \frac{8}{3} = \frac{8\pi}{3} \ .
\end{aligned}$$