Solutions: quiz 11, discussion section 11am

Math 226, Fall 2019, Prof. Mazel-Gee

1. We compute that

$$\int_C \mathbf{F} \bullet d\mathbf{r} = \int_{t=0}^{t=1} \mathbf{F}(\mathbf{r}(t)) \bullet \mathbf{r}'(t)\ dt = \int_{t=0}^{t=1} \langle (2t^2)\cdot(3t), 2(3t)^2 \rangle \bullet \langle 4t, 3 \rangle\ dt = \int_{t=0}^{t=1} (24t^4 + 54t^2)\ dt$$
$$= \left(\frac{24}{5}t^5 + 18t^3 \right)\Bigg|_{t=0}^{t=1} = \frac{24}{5} - 18 = -\frac{66}{5}\ .$$

2. We begin by observing that $\mathbf{F}$ is defined on all of $\mathbb{R}^2$, which is open and simply-connected. We observe moreover that $\frac{\partial}{\partial y}(e^x \sin y) = e^x \cos y = \frac{\partial}{\partial x}(e^x \cos y)$. So, $\mathbf{F}$ is conservative. A potential function $f(x,y)$ must have that $f_x = e^x \sin y$, which implies that $f(x,y) = e^x \sin y + C_1(y)$; it must also have that $f_y = e^x \cos y$, which implies that $f(x,y) = e^x \sin y + C_2(x)$; together these imply that $f(x,y) = e^x \sin y + C$.