Solutions: quiz 3, discussion section 10am

Math 226, Fall 2019, Prof. Mazel-Gee

1. For an arbitrary point $(x, y, z) \in \mathbb{R}^3$, its distance to the point $(0, 4, 0)$ is $\sqrt{x^2 + (y-4)^2 + z^2}$ while its distance to the xy-plane is $|y|$. Since these quantities are both nonnegative, equating them is equivalent to equating their squares, and so we arrive at the equation

$$x^2 + (y-4)^2 + z^2 = y^2 \ ,$$

which simplifies to

$$y = \frac{x^2}{8} + \frac{z^2}{8} + 2 \ .$$

This is the equation of a (circular, not merely elliptic) paraboloid opening in the positive-y direction with vertex $(0, 2, 0)$.

2. We begin by computing that

$$f'(t) = \mathbf{u}'(t) \bullet \mathbf{v}(t) + \mathbf{u}(t) \bullet \mathbf{v}'(t) \ ,$$

so that

$$f'(0) = \mathbf{u}'(t) \bullet \mathbf{v}(t) + \mathbf{u}(t) \bullet \mathbf{v}'(t) \ .$$

We also compute that $\mathbf{u}'(t) = \langle e^t, 2e^{2t}, 3e^{3t} \rangle$, so that $\mathbf{u}(0) = \langle 1, 1, 1 \rangle$ and $\mathbf{u}'(0) = \langle 1, 2, 3 \rangle$. So we find that

$$f'(0) = \langle 1, 2, 3 \rangle \bullet \langle 0, 1, 0 \rangle + \langle 1, 1, 1 \rangle \bullet \langle 1, -1, 1 \rangle = 3 \ .$$