

Name: _____

- No electronic devices are allowed. You must show your work to obtain credit.
- You may use the back if necessary. Please indicate clearly if you do so.

1. (5 points) If $z = f(x, y)$, where $x = r \cos \theta$ and $y = r \sin \theta$, show that when $r \neq 0$,

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2.$$

(Possible hint: use chain rule to find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ in terms of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, then compare the right-hand side with the left-hand side)

2. (5 points) Find the gradient vector at point (x_0, y_0) of the function

$$f(x, y) = \frac{xy}{x + y},$$

then find the directional derivative of $f(x, y)$ at point $(1, 2)$ in the direction of the vector $\langle 2, 1 \rangle$.