Solutions: quiz 8, discussion section 11am

Math 226, Fall 2019, Prof. Mazel-Gee

1. Note that R is a polar rectangle for which $\theta \in [\frac{3\pi}{4}, \pi]$ and $r \in [0,1]$. Therefore, we have that

$$\iint_R (x-y)\ dA = \int_{\theta=\frac{3\pi}{4}}^{\theta=\pi} \int_{r=0}^{r=1} (r\cos\theta - r\sin\theta)\cdot r\ dr\ d\theta = \left(\int_{\theta=\frac{3\pi}{4}}^{\theta=\pi} (\cos\theta - \sin\theta)\ d\theta \right) \cdot \left(\int_{r=0}^{r=1} r^2\ dr \right)$$
$$= (\sin\theta + \cos\theta)\Big|_{\theta=\frac{3\pi}{4}}^{\theta=\pi} \cdot \left(\frac{1}{3} r^3 \right)\Bigg|_{r=0}^{r=1} = \left((0 + (-1)) - \left(\frac{1}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}} \right) \right) \right) \cdot \frac{1}{3} = -\frac{1}{3}\ .$$

2. The indicated region R is of type II, where $y \in [-1,1]$ and $y^2 \leq x \leq 1$. Because the function $f(x,y) = 1 + xy$, we may apply Fubini's theorem to compute the volume to be

$$\iint_R (1+xy)\ dA = \int_{y=-1}^{y=1} \int_{x=y^2}^{x=1} (1+xy)\ dx\ dy = \int_{y=-1}^{y=1} \left(x + \frac{y}{2} x^2 \right)\Big|_{x=y^2}^{x=1}\ dx$$
$$= \int_{y=-1}^{y=1} \left(\left(1 + \frac{y}{2} \right) - \left(y^2 + \frac{y^3}{2} \right) \right)\ dy = \left(y + \frac{y^2}{4} - \frac{y^3}{3} - \frac{y^4}{8} \right)\Bigg|_{y=-1}^{y=1}$$
$$= \frac{2}{3} - \left(-\frac{2}{3} \right) = \frac{4}{3}\ .$$