Solutions: quiz 9, discussion section 10am

Math 226, Fall 2019, Prof. Mazel-Gee

1. We compute the mass m to be

$$\iint_D \rho(x,y)\ dA = \int_{x=0}^{x=L} \left(\int_{y=0}^{y=\sin(\pi x/L)} y\ dy \right) dx = \int_{x=0}^{x=L} \left(\frac{1}{2} y^2 \right)\Big|_{y=0}^{y=\sin(\pi x/L)} dx$$
$$= \int_{x=0}^{x=L} \frac{1}{2} \sin^2(\pi x/L)\ dx = \int_{u=0}^{u=\pi} \frac{L}{2\pi} \sin^2 u\ du = \int_{u=0}^{u=\pi} \frac{L}{4\pi}(1-\cos 2u)\ du$$
$$= \frac{L}{4\pi} \left(u - \frac{1}{2}\sin 2u \right)\Big|_{u=0}^{u=\pi} = \frac{L}{4}\ .$$

2. Writing these paraboloids in cylindrical coordinates as $z = r^2$ and $z = 2 - r^2$, we see that their intersection has $r = 1$. From here, using cylindrical coordinates we compute the volume of this region E to be

$$\iiint_E 1\ dV = \int_{\theta=0}^{\theta=2\pi} \left(\int_{r=0}^{r=1} \left(\int_{z=r^2}^{z=2-r^2} r\ dz \right) dr \right) d\theta = 2\pi \cdot \int_{r=0}^{r=1} (rz)|_{z=r^2}^{z=2-r^2}\ dr$$
$$= 4\pi \cdot \int_{r=0}^{r=1} (r - r^3)\ dr = 4\pi \cdot \left(\frac{1}{2}r^2 - \frac{1}{4}r^4 \right)\Big|_{r=0}^{r=1} = \pi\ .$$