SYLLABUS
FUNDAMENTAL CONCEPTS OF MODERN ALGEBRA (MATH 410)
FALL 2019

Contents

1. General information

1.1. Instructor.
name: Prof. Mazel-Gee
email: aaron.mazelgee@usc.edu
office hours: Mon. 10am-12pm and Wed. 2-3pm in KAP 438D[1]

1.2. Teaching assistant (TA).
name: Danjoseph Quijada
email: dquijada@usc.edu
office hours: Wed. 5-6pm and Thurs. 11:30am-1:30pm in KAP 263

1.3. Course logistics.
lectures: MWF at 1-1:50pm in VKC 210
discussion sections: Th at 2-3:50pm in WPH 107
textbook: *Introduction to Abstract Algebra*, Nicholson (4th ed.)
course website: etale.site/teaching/f19-410/
Blackboard: yes

1.4. Grades. Your final grade in the course will be determined by your performance on homework and exams (two midterms and one final). Your scores on these will be combined into a single overall *raw score*, according to the following weights:

homework: 25%, midterms: $2\times 20\%$, final exam: 35%.

last updated: September 9, 2019

[1]If all of my office hours conflict with your schedule, I am also available to meet by appointment.

At the end of the semester, letter grades will be assigned on a curve based on these raw scores, with the top quartile receiving grades in the A-range and the second quartile receiving grades in the B-range (so the median grade will be the cutoff between a B- and a C+).[2] Over the course of the semester, I will periodically compute your raw scores up to that point and inform you of the current grade distribution, to give you a sense of where you stand.

1.5. **Homework.** Homework will be due at the beginning of discussion section each Thursday.[3] Homework must be written in LaTeX (see §2.2 below) and submitted electronically. Late homework will not be accepted. In order to account for unexpected circumstances, your lowest two homework scores will be dropped.

1.6. **Exams.** The exam schedule is as follows:

midterm 1: Wed. 9/25, midterm 2: Wed. 10/30, final exam: Wed. 12/18 at 11am-1pm.[4]

There are no make-up exams.

2. Course description

2.1. **Course content.** Broadly speaking, the field of ***algebra*** – also called "modern algebra" or "abstract algebra" – is the mathematics of *symmetry*. It arose during the 1800s through a combination of investigations relating to number theory, geometry, and the search for general formulas for solving polynomials.[5] Today, algebra plays a central and pervasive role in research-level mathematics, and also appears in scientific disciplines such as chemistry and theoretical physics.

A driving force in algebra, which is largely the reason for its great power and flexibility, is the passage from specific questions to general abstract theory. A famous example of this phenomenon is the proof of Fermat's last theorem: the nonexistence of nontrivial integer solutions $(a, b, c) \in \mathbb{Z}^3$ to the equation

$$a^n + b^n = c^n$$

for any integer $n \geq 3$.[6] Another is the proof of the insolvability of the quintic: the nonexistence of a general formula for solving degree-5 polynomials (akin to the quadratic equation

[2]The curve will include students who have dropped the course (who will presumably lie towards the bottom of the rankings).

[3]This includes the first week, although the homework assignment will be shorter than usual. Also, homework will be due on Thursday 10/17 at 2pm (even though there will not be discussion section due to fall recess).

[4]The midterms will be given in class, while the final exam will be given in a location that is TBD.

[5]In fact, a revolutionary idea that arguably made Grothendieck the most influential mathematician of the 1900s is that *these are all instances of the same thing*. More specifically, Grothendieck invented a robust theory of *algebraic geometry* that brought algebraic (and in particular, number-theoretic) questions into the purview of the methods and tools of geometry.

[6]Here, "nontrivial" simply means that $abc \neq 0$.

for solving degree-2 polynomials). Note that these are both assertions of *impossibility*. It would be impossible to verify either of them directly, because one would have to verify the failure of infinitely many possible solutions (triples of integers $(a, b, c) \in \mathbb{Z}^3$ or algebraic formulas in the coefficients of the general quintic equation $a_0 + a_1x + \cdots + a_5x^5 = 0$); and neither a human nor a computer is capable of directly checking infinitely many cases. Thus, it is only through more sophisticated techniques – the "general abstract theory" referred to above – that one can prove such assertions of impossibility. And indeed, both of these mathematical achievements rely on rich and extensive algebraic developments.[7]

In this course, we will likewise study specific examples of the fundamental objects of algebra – groups, rings, and fields – appearing in modular arithmetic before proceeding to their general abstract theory. For example, we will learn how to answer the question "What is the remainder when 4^{119} is divided by 7?" in a systematic way, and we will prove that for any prime number p and any integer n the integer $n^p - n$ is divisible by p.

The specific aspects of the general theory of groups, rings, and fields that we study will be determined as we go. But in any case, an overarching goal of the course will be for you to develop and hone your ability to write, comprehend, and analyze proofs. **A proof is a logical argument that inexorably compels the reader to believe the given assertion.**

Note that proofs entail both *logical thinking* and *communication*. While formal proofs themselves primarily fall within the realm of mathematics,[8] the ability to understand what does and does not constitute a proof – that is, an airtight logical argument – is a critically important and indefinitely useful life skill. Because this will be a new experience for many of you, we will begin with a review of proofs.

2.2. **LaTeX.** As indicated in § 1.5, you will be required to write your homework in LaTeX (pronounced "LAH-teck"). This is a versatile and easy-to-use typesetting system that produces beautiful mathematical documents such as the one you are reading now.

There are a number of reasons for this requirement. Most of all, writing your homework in LaTeX will help you improve your proof-writing skills: it is much easier to see whether your argument is logically sound when it is laid out cleanly and clearly in complete sentences (as opposed to being e.g. just a long handwritten string of logical symbols). And then, if you are

[7] The proof of the insolvability of the quintic is relatively approachable, and is not so far beyond the scope of this course. By contrast, the proof of Fermat's last theorem is incredibly complex; shortly after it appeared (in 1994, a mere 358 years after it was first conjectured), 28 experts from around the world assembled for a 10-day conference with the sole purpose of fully understanding it, and they collectively wrote a 500-page book in an attempt to make it accessible to graduate students studying number theory.

[8] Proofs also appear in physics and computer science. Lawyers also like to talk about proof, but their standards are necessarily far lower than what is being discussed here. For example, here in the U.S., in order to convict somebody of a crime, a prosecutor must "prove beyond a reasonable doubt" that they are guilty. In mathematics, we are concerned with proofs that are beyond *any* doubt at all!

anything like me, you will also gain a newfound sense of pride, satisfaction, and ownership from discovering mathematical truths and recording them in an aesthetically appealing way. Lastly, LaTeX has become the industry standard throughout most quantitative academic fields, and so this experience will be an asset to many of you in your future careers.

Here is a document outlining some resources to help you get started writing in LaTeX.

2.3. **Lectures, reading, and discussion section.** The material covered in the lectures will closely follow that covered in the book. However, lectures will often present the material in a slightly different way, or at least with different examples. Thus, the lectures and the book should complement each other; you should expect both to attend all lectures and to read the book diligently in order to succeed in this course. Meanwhile, the discussion sections – led by your TA – will give you the opportunity to focus on some of the more difficult concepts that are covered in lecture, as well as to actively engage with the material along with your fellow classmates.

3. Comments, suggestions, and resources

3.1. **General comments and suggestions.** Abstract algebra is a uniquely challenging course in the undergraduate math curriculum, because one does *not* come to it with any prior intuition. For example, there is no way to guess the answer to the question "How many groups are there that have exactly 20 elements?" based on concepts you have seen in other courses. This sits in contrast with a course such as real analysis (Math 425A here at USC), in which one studies already-familiar mathematical concepts such as real numbers, limits, integrals, and so on. Hence, regardless of your mathematical background, you should expect to spend at least as much time on this course as you have ever spent on a math class, and quite possibly more. The only way to really get to understand new and challenging mathematical concepts is to spend *a lot* of time with them.

In addition, I encourage you to take advantage of the resources available to you that are described below, *throughout* the semester (i.e. not just in the days leading up to exams). These resources can make the time that you do spend on this course much more effective, especially if you use them wisely.

I like to compare mathematics with running. The only way to get really good at running is to do a lot of it. Moreover, you have to do a lot of it *for a long time*. You wouldn't start training for a marathon just a few days before – that'd be way too late! Similarly, you should not expect to do well on the exams if you don't stay on top of the material throughout the course.

Even if the introductory material on proofs is familiar to you, I urge you to pay close attention nonetheless: it is always possible to become better acquainted with these ideas, and they will be of utmost importance throughout the semester.

With all of this said, it is also worth mentioning that this is a very fun and rewarding course, which should be quite different from what you have seen in other courses.

3.2. **Attribution.** Whenever you receive help on any of your homework problems (such as in the ways described below – office hours, Math Center, collaboration, etc.), you must indicate this on your homework, either at the beginningof the entire document or as a footnote to specific problems. For instance, at the top of your homework you might write "I worked on my homework with Alice and Bob.", or as a footnote to problem 17 you might write "I found a hint/solution to this at `http://mathhelp4u.blogspot.com`.". *It is considered cheating if you do not provide such attributions*, and you may face consequences accordingly.

3.3. **Office hours.** Almost certainly, the most valuable and effective resource for you will be my office hours. You should know, however, that they are made far more valuable if you spend time with the material *before* coming to office hours.

Compare this to learning a musical instrument. There are two aspects to this: taking lessons from a teacher, and practicing on your own. These are both very helpful. A teacher can give you suggestions for improving your technique as well as broader guidance in your musical studies. But this is useless if you don't practice on your own as well.

On the other hand, you should not feel that you need to come to office hours with specific questions prepared. When we are confused, often the hardest part of becoming un-confused is just articulating exactly where our confusion lies. Having spent many years learning and teaching (and researching) math, I will very likely be able to help you identify and resolve your confusion.

Note that if you do not work on the homework before coming to office hours, *you won't even know whether you're confused*. You will still be welcome to come to office hours in this case, but this will make that time much less effective for you.

3.4. **Math Center.** The Math Center (KAP 263) is an amazing resource: in essence, it is a centralized location where students can come get help from TAs, that is open pretty much all day every (week)day.[9] The Math Center schedule will be fixed by the end of the second week of the semester, at which time it will be available on the Math Center website. Your TA holds office hours there, as do all TAs for core math classes. While the other TAs will be prioritizing their own students, most of them will be happy to help you if they have time to do so. In any case, you should feel free to use the Math Center as a place to meet and study with your fellow students.

3.5. **Collaboration.** Collaboration with your fellow students can be a very effective way of learning, and is highly encouraged. Often, we may think we understand something very

[9]Towards helping you appreciate this, it's worth mentioning that it is fairly unique to USC. For instance, at my undergrad institution we had a similar sort of math center, but it was only open 8 hours per week.

well, but in trying to explain it to somebody else we find the gaps in our understanding.[10] However, *everyone must write up their own solutions separately*: you may not copy each other's work. Moreover, as described in §3.2, you must indicate on your homework if you worked with anyone else.

3.6. **Extra problems.** You should view the homework as the *minimum* set of problems that you should work through: it is highly encouraged to work through other problems as well. The problems in the book are roughly organized in increasing order of difficulty. So for instance, if you are struggling with the concepts, you should attempt some easy problems to try and figure out what you're missing. And if you are studying for an exam and want to really challenge your grasp of the material, you should test yourself on some hard problems.

I believe that it can be extremely worthwhile to *revisit the same problems multiple times*, even at the expense of seeing fewer problems overall. In preparing for exams, students often make a point of doing lots of different problems, with the idea that this will make them more likely to have already seen whatever problems end up on the exam. However, it's not very useful to have seen a problem if you don't have a solid understanding of how to solve it! Understanding is not binary: there are degrees to it, and it can ebb and flow with time. In my experience, it is generally a better idea to become very familiar with a smaller list of problems by doing each of them many times.[11]

3.7. **The book.** Reading a math textbook is very different from reading a novel. You should expect to have to read things slowly, and often multiple times, in order to fully understand what is being expressed. If you are getting confused, you may even want to take a break from reading the book and come back to it again later. Keep in mind that the book was written *for you*: by and large, it is as clear as it can possibly be. The reason that it is difficult to read is because the material itself is difficult. And that's okay – it's as it should be.

3.8. **The internet.** There are tons of resources out there, and some of them are very good.

4. Miscellanea

4.1. **Blackboard.** Homework assignments, readings, and announcements concerning the course will be posted to Blackboard; it is your responsibility to check there frequently. Your

[10] There was some research that came out about this recently. Subjects were asked to rate their understanding of some everyday mechanism (e.g. a ballpoint pen or the flushing mechanism of a toilet). Then, they were asked to explain it. Then, they were asked to rate their understanding again. As you might imagine, their own estimation of their understanding dropped significantly from the first round of rating to the second.

[11] This level of familiarity will also allow you to see deeper similarities between problems, which you may not have appreciated if you had only skimmed through all the different problems.

scores will also be recorded on Blackboard; it is your responsibility to check that your scores are recorded correctly.

4.2. **Course website.** As the course progresses, various materials will become available at the course website.

4.3. **Communication.** In general, you are always welcome to email me and/or the TA about any issue regarding the course. However, please check the syllabus and the Blackboard announcements before emailing, in case your question has already been answered.

4.4. **Calculators.** We will not be using calculators in this course.

4.5. **Classroom conduct.** Please remember that you and your classmates are here to learn, and refrain from disruptive or otherwise inconsiderate behavior. In particular, cell phones are to be turned off and kept out of sight during class: even if you yourself are able to multitask or already understand the material, using your phone will likely be distracting to your fellow classmates. If I see you on your phone, I will ask you to either put it away immediately or leave class until you are done with it.

4.6. **DSP accommodations.** Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. Please ensure that DSP delivers its letter of verification to me as soon as possible, to give me enough time to make suitable arrangements. *I must receive your letter at least two weeks before an exam in order to guarantee that you will receive the corresponding accommodations.* You can find information about DSP (location, hours, contact, etc.) here.

4.7. **Academic integrity.** You must abide by the university policies on academic integrity, which you can review here. In essence, these policies require you to be honest. So, please: be honest.

4.8. **Revisions to this syllabus.** This syllabus is subject to minor changes throughout the semester, as needed. In the interest of transparency, all versions will remain available on the course website.