SYLLABUS
ALGEBRAIC AND DIFFERENTIAL TOPOLOGY (MATH 151A)
FALL 2020

Contents

1. General information

1.1. Instructor.

name: Aaron Mazel-Gee
email: mazelgee@caltech.edu
office hour: 1:30-2:30pm on Fridays[1]
pronouns: he/him

1.2. Teaching assistant (TA).

name: Nathan Sagman
email: nsagman@caltech.edu
office hour: 5-6pm on Mondays
pronouns: he/him

1.3. Course logistics.

All interactions will be conducted via Zoom. Links will be made available in Canvas.
lectures: MWF at 10-11am
textbooks: *Algebraic Topology*, Hatcher; *A Concise Course in Algebraic Topology*, May
course website: etale.site/teaching/f20/
Canvas: yes, lightly (see §4.2)

last updated: October 9, 2020

[1]If my office hour conflicts with your schedule, I am also available to meet by appointment.

1.4. **Homework and grades.**

Homework will generally be assigned weekly. It will be due by the beginning of lecture (i.e. at 10am) on the due date, which will generally be Wednesdays. Your final grade in the course will be determined by your performance on the homework.

Please read the brief document Guidelines for Good Mathematical Writing, and adhere to its suggestions to the greatest extent possible.

You are encouraged to write your homework in LaTeX (pronounced "LAH-teck"). This is a versatile and easy-to-use typesetting system, which is the industry standard in mathematics and adjacent fields. Some resources to help you get started writing in LaTeX are outlined in §4.5.

You will submit your homework by email directly to your TA. If you write your homework in LaTeX, you may simply submit the pdf. Alternatively, you may write your homework out by hand and submit photos of it, assembled as a single pdf (e.g. using an online photo-to-pdf converter).

Some homework problems may ask you to draw pictures. In such cases, you may either embed photos into your LaTeX file or simply attach separate photos to your email.

In order to account for unexpected circumstances, you are automatically granted a cumulative deadline extension of 72 hours.

2. Course description

Broadly speaking, the fundamental aim of topology is to understand manifolds. What we generally mean by this is that we attach *invariants* to manifolds. The purpose of doing so is twofold:

(1) to distinguish between manifolds based on their values; and
(2) to encode geometric (or at least topological) information.

As a basic example of (1), the Euler characteristic can distinguish between the torus and the sphere: we have the inequality

$$\chi(T^2) = 0 \neq 2 = \chi(S^2) \ ,$$

and so it must be the case that $T^2 \not\cong S^2$. However, from the point of (2), the Euler characteristic is rather deficient: it does not admit a uniform geometric description, and it tells us little about the manifold itself – for instance, it cannot tell us its number of path components or its dimension.

This situation may be remedied by passing from the Euler characteristic to *homology*. Given a manifold M, its homology is a sequence of vector spaces $\{H_n(M)\}_{n\geq 0}$ with the following two features.

(1) For each $n \geq 0$, the dimension of the vector space $H_n(M)$ reflects the number of "n-dimensional holes" in M.

(2) We can recover the Euler characteristic of M as the alternating sum

$$\chi(M) = \sum_{n \geq 0} (-1)^n \dim(H_n(M)) .$$[2]

As a result of (2), homology is a strictly stronger invariant than the Euler characteristic. Of course, it is also necessarily at least as difficult to compute. However, as it turns out, homology is not much harder to compute than the Euler characteristic. It also enjoys an additional feature known as *functoriality*, which is helpful both for computation and because it makes homology into a more sensitive invariant. Hence, homology provides a striking improvement over the Euler characteristic as an invariant of manifolds.

An important and recurring theme in topology is the interplay between differential and algebraic techniques. For example, homology can be described in at least six ways:

(1-2) in differential topology, using *de Rham theory* or *Morse theory*;

(3) in differential geometry, using *Hodge theory*; or

(4-6) in algebraic topology, using *cellular*, *simplicial*, or *singular* homology.

These various perspectives synergize both computationally and conceptually. In particular, the various constructions of homology that are native to algebraic topology each have their own advantages and distadvantages: cellular and simplicial homology are directly computable, whereas singular homology is functorial.

In this course, we will study some basic algebraic invariants such as the fundamental group, the fundamental groupoid, and (cellular and singular) homology. If time permits, we will also cover more advanced topics (e.g. generalized homology theories, homological/homotopical algebra, and the Dold–Thom theorem). Throughout, we'll try to balance geometric intuition with algebraic machinery. This is why we will be using two references: Hatcher supplies a conversational treatment of the former, while May supplies an efficient treatment of the latter.

The material covered in the lectures will generally be a blend of the material contained in the two books, although there will also be digressions that are not contained in either. Thus, the lectures and the books should complement each other. In order to succeed in this course, you should expect both to attend (or asynchronously view) all lectures and to regularly read one or both books (depending on the material as well as on your own taste).

3. Resources

Collaboration with your fellow students can be a very effective way of learning, and is highly encouraged. Often, we may think we understand something very well, but in trying to explain it to somebody else we find the gaps in our understanding.[3] However, *everyone must*

[2] This sum will always be finite, because $H_n(M) = 0$ whenever $n > \dim(M)$.

[3] There was some research that came out about this recently. Subjects were asked to rate their understanding of some everyday mechanism (e.g. a ballpoint pen or the flushing mechanism of a toilet). Then,

write up their own solutions separately: you may not copy each other's work. Moreover, you must indicate any help that you received on your homework (e.g. from office hours, websites, collaboration with other students, etc.), either at the beginning of the entire document or as a footnote or marginalium to specific problems. For instance, at the top of your homework you might write "I worked on my homework with Alice and Bob.", or as a footnote to Problem N you might write "I found a hint/solution to this at `http://mathhelp4u.blogspot.com`.".

As a guideline for this collaboration policy, keep in mind that **you should be able to reproduce any solution you hand in without assistance**.

Here is a list of allowed and disallowed resources. This is intended to be reasonably comprehensive, but the fact that something is not on this list does *not* mean that it is permitted. If you are ever in doubt about whether something is allowed, it is **your responsibility** to ask.

you may consult:	
course textbooks	yes
other books	yes
solution manuals	no
internet	yes
notes from lecture (yours or others')	yes
course handouts	yes
your past homework	yes
homework and exams solutions posted on webpage	yes
homework and exams from previous years	no
communications (e.g. emails) from instructor or TA	yes
you may:	
discuss problems with others	yes
look at communal materials (e.g. (e-)whiteboard) when writing up solutions	yes
look at written work of others	no
post about problems online (e.g. at math.stackexchange)	yes
for computational aid, you may use:	
calculator	yes[4]
computer	yes[4]

they were asked to explain it. Then, they were asked to rate their understanding again. As you might imagine, their own estimation of their understanding dropped significantly from the first round of rating to the second.

[4] It is not expected that computational aids will be helpful in this course (except possibly for visualization purposes). Nevertheless, you may use a calculator or computer while doing homework. However, you may not refer to this as a justification of your work. For example, "by Mathematica" is not an acceptable justification by itself.

4. Miscellanea

4.1. **Attendance and participation.** While they do not directly affect your grade, I would very much appreciate your regular attendance and participation in lectures. In my own experience, mathematics is most exciting and most rewarding when it is pursued as an interpersonal endeavor: as an interesting and unique way to interact with other human beings.

Relatedly, I would also appreciate you leaving your video on during lectures. This will help me gauge your comprehension and attention. You will also likely find it useful as a way of keeping yourself accountable, so that you are less likely to distract yourself (and therefore more likely to ask a question!) if and when you get lost.

4.2. **Canvas.** Much of the relevant information for this course will be posted on the course website. However, we will use Canvas for a few purposes, e.g. for any material that may be considered as sensitive or unpolished – lecture video recordings, lecture notes, Zoom links, chat, discussion boards, grades, etc.

4.3. **Students with documented disabilities.** If you need an academic accommodation based on the impact of a disability, you must initiate the request with Caltech Accessibility Services for Students (CASS). Professional staff will evaluate your request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for the quarter in which the request is being made. In this case, you should contact CASS as soon as possible, since timely notice is needed to coordinate accommodations. Undergraduate students should contact Dr. Lesley Nye, Associate Dean of Undergraduate Students (administrative contact: Beth Larranaga) and graduate students should contact Dr. Kate McAnulty, Associate Dean of Graduate Studies (administrative contact: Jacob Dalton).

4.4. **Academic integrity.** You must abide by Caltech's honor code: *No member of the Caltech community shall take unfair advantage of any other member of the Caltech community.* General guidelines for how this applies to plagiarism are given here. Specific guidelines for this course are described in §3.

4.5. **LaTeX resources.** LaTeX code is stored as plain text, and therefore can be written and saved with the filename extension `txt` (e.g. as `my-LaTeX-code.txt`). This raw code can then be *compiled* into a `pdf` file (such as the one you are reading now). You can compile LaTeX code directly on your computer after a somewhat lengthy download/installation process. Alternatively, you can write and compile LaTeX code online through the website Overleaf. I would recommend the latter option, especially if you are having difficulty with the former option (which is somewhat nontrivial).

A simple template that you can use to get started on your homework is available here; when compiled, that LaTeX code produces this file. Of course, you are also welcome to start from scratch. Along with occasional online searches, that template should suffice to get you started.[5] However, if you would like a further introduction to LaTeX you can check out some of the instructions provided by Overleaf: this shorter one or this longer one.

Of course, there are many online LaTeX resources. In particular, there are many "crib sheets", see e.g. here and here. Another great resource is Detexify: if you draw a mathematical symbol, it will attempt to suggest how to write it in LaTeX.

4.6. **Revisions to this syllabus.** This syllabus is subject to minor changes throughout the semester, as needed. In the interest of transparency, all versions will remain available on the course website.

[5]The diagram is drawn using the `tikz-cd` package, more information about which can be found here.